



Printed Pages : 7

TAS-204

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9929

Roll No.

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B. Tech.

(SEM. II) EXAMINATION, 2007-08

MATHEMATICS - II

Time : 3 Hours]

[Total Marks : 100

- Note :**
- (1) Attempt *all* questions.
 - (2) All questions carry *equal* marks.
 - (3) In case of numerical problems assume data wherever not provided.
 - (4) Be precise in your answer.

1 Attempt any **four** parts of the following : **5×4=20**

(a) Solve $\left(y + \sqrt{x^2 + y^2}\right) dx - x dy = 0,$

$y(1) = 0.$

(b) Solve the following differential equation :

$(\cos x - x \cos y) dy - (\sin y + y \sin x) dx = 0$

(c) Find the solution of following differential equation :



$$(D^2 - 4D - 5)y = e^{2x} + 3 \cos(4x + 3)$$

$$\text{where } D = \frac{d}{dx}$$

- (d) Solve the following simultaneous differential equations :

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0$$

Also show that $x = y = 0$ when $t = 0$.

- (e) Solve by method of variation of parameters

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \log x.$$

- (f) An inductance (L) of 2.0 H and a resistance (R) of 20Ω are connected in series with an e.m.f. E volt. If the current (i) is zero, when $t = 0$, find the current (i) at the end of 0.01 second if $E = 100 \text{ V}$, using the following differential equation :

$$L \frac{di}{dt} + iR = E.$$



2 Attempt any **four** parts of the following : **5×4=20**

(a) Using Laplace transform, evaluate

$$\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt.$$

(b) State second shifting theorem for Laplace transform and hence find the Laplace transform of the following function

$$F(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$$

(c) Using convolution theorem, find the inverse

Laplace transform of the following $\frac{s}{(s^2 + a^2)^3}$.

(d) Solve the following simultaneous differential equations by Laplace transform

$$\frac{dx}{dt} + 4 \frac{dy}{dt} - y = 0$$

$$\frac{dx}{dt} + 2y = e^{-t}$$

with condition $x(0) = y(0) = 0$.



- (e) Solve the following differential equation using

Laplace transform $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 5y = e^{-x} \sin x$

where $y(0) = 0, \left(\frac{dy}{dx} \right)_{x=0} = 1.$

- (f) Using unit step function, find the Laplace transform of :

(i) $(t-1)^2 \cdot u(t-1)$

(ii) $\sin t \cdot u(t-\pi).$

3 Attempt any **two** parts of the following : **10×2=20**

- (a) Solve the following differential equation in series :

$$2x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + (x-5)y = 0.$$

- (b) Show that

$$(1) \int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{n+1}$$



$$(2) \quad P_{2n}(0) = \frac{(-1)^n (2n)!}{2^{2n} (n!)^2}$$

where $P_n(x)$ is the Legendre polynomial of degree n .

(c) Show that

$$(1) \quad J_2^1(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$$

$$(2) \quad J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$$

where $J_n(x)$ is the Bessel's polynomial of degree n and dash denote the differentiation.

4 Attempt any **two** parts of the following : 10×2=20

(a) Obtain a Fourier series to represent $f(x) = x^2$

in the interval $(0, 2\pi)$ and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

(b) Examine whether the function $f(x) = x \sin x$ is even or odd. Hence expand it in the form of Fourier series in the interval $(-\pi, \pi)$.



(c) Solve the following partial differential equations :

$$(1) \quad x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$$

$$(2) \quad \frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y$$

$$\text{where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

5 Attempt any **two** parts of the following : **10×2=20**

(a) Solve the following equation by the method of separation of variables :

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y} \text{ where } u(0, y) = 8 e^{-3y}.$$

(b) Solve the following Laplace equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle with $u(0, y) = 0$, $u(a, y) = 0$,

$u(x, b) = 0$ and $u(x, 0) = f(x)$ along

x -axis.



(c) Assuming the resistance of wire (R) and conductance to ground (C_P) are negligible, find the voltage $v(x, t)$ and current $i(x, t)$ in a transmission line of length l , t seconds after the ends are suddenly grounded. The initial conditions are $v(x, 0) = v_0 \sin\left(\frac{\pi x}{l}\right)$ and

$$i(x, 0) = i_0.$$

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